

FORMULE di BISEZIONE

$\sin 15^\circ \cos 75^\circ \dots \sin 36^\circ$

$$\begin{aligned}\cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 1 - 2\sin^2 \alpha = \\ &= 2\cos^2 \alpha - 1\end{aligned}$$

$$\cos \underbrace{2\alpha}_\beta = 1 - 2\sin^2 \underbrace{\alpha}_{\beta/2}$$

$$\cos \beta = 1 - 2\sin^2 \frac{\beta}{2}$$

$$2\sin^2 \frac{\beta}{2} = 1 - \cos \beta$$

$$\sin^2 \frac{\beta}{2} = \frac{1 - \cos \beta}{2}$$

$$\left[\sin \frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos \beta}{2}} \right]$$

$$\sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} =$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\sqrt{a + \sqrt{b}} \quad \sqrt{a - \sqrt{b}} \quad a^2 - b = k^2$$

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$\sqrt{2 - \sqrt{3}} = \sqrt{\frac{2 + \sqrt{4 - 3}}{2}} - \sqrt{\frac{2 - \sqrt{4 - 3}}{2}}$$

$$= \sqrt{\frac{2+1}{2}} - \sqrt{\frac{2-1}{2}} = \sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}} =$$

$$= \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{2}$$

$$\sqrt{2 - \sqrt{3}} = \sqrt{\frac{1}{2} + \frac{3}{2} - 2 \cdot \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{3}{2}}}$$

$$= \sqrt{\left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}\right)^2} = \sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}$$

$$\left(\operatorname{tg} \frac{\beta}{2}\right) = \pm \sqrt{\frac{1 - \cos \beta}{1 + \cos \beta}}$$

$$\sqrt{\frac{1 - \cos \beta}{1 + \cos \beta}} \cdot \frac{\sqrt{1 - \cos \beta}}{\sqrt{1 - \cos \beta}} \quad \cos \beta \neq 1 \quad \beta \neq 2k\pi$$

$$= \sqrt{\frac{(1-\cos\beta)^2}{\sec^2\beta}} = \boxed{\frac{1-\cos\beta}{\sec\beta}}$$

$$\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} \cdot \frac{\sqrt{1+\cos\beta}}{\sqrt{1+\cos\beta}} \quad \cos\beta \neq -1 \quad \beta \neq \pi + 2k\pi$$

$$\sqrt{\frac{\sec^2\beta}{(1+\cos\beta)^2}} = \boxed{\frac{\sec\beta}{1+\cos\beta}}$$

β	$\beta/2$	$\tan \beta/2$	$\sec\beta$
I	I	+	+
II	I	+	+
III	II	-	-
IV	II	-	-

$$\cos 72^\circ = \frac{\sqrt{5}-1}{4}$$

$$\sec 36^\circ = + \sqrt{\frac{1 - \frac{\sqrt{5}-1}{4}}{2}}$$

$$= \sqrt{\frac{4 - \sqrt{5} + 1}{8}}$$

$$= \sqrt{\frac{5 - \sqrt{5}}{8}}$$

$$= \underline{\underline{\sqrt{5-\sqrt{5}}}} \cdot \underline{\underline{\sqrt{2}}} = \underline{\underline{\sqrt{2(5-\sqrt{5})}}}$$

$$2\sqrt{2} \quad \sqrt{2} \quad 4$$

$$= \frac{\sqrt{10-2\sqrt{5}}}{4}$$