

## PROSTAFERESI

$$\begin{aligned} \text{sen } \alpha \pm \text{sen } \beta \\ \cos \alpha \pm \cos \beta \end{aligned}$$

$$\begin{aligned} \text{sen}(\alpha + \beta) &= \text{sen } \alpha \cos \beta + \cos \alpha \text{sen } \beta \\ \text{sen}(\alpha - \beta) &= \text{sen } \alpha \cos \beta - \cos \alpha \text{sen } \beta \end{aligned}$$

$$\text{sen}(\alpha + \beta) + \text{sen}(\alpha - \beta) = 2 \text{sen } \alpha \cos \beta$$

$$\begin{cases} \alpha + \beta = p \\ \alpha - \beta = q \end{cases}$$
$$\frac{2\alpha}{2} = \frac{p+q}{2} \quad \beta = \frac{p-q}{2}$$
$$\alpha = \frac{p+q}{2} \quad \beta = \frac{p-q}{2}$$

$$\rightarrow \text{sen } p + \text{sen } q = 2 \text{sen } \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\begin{aligned} \text{sen } 15^\circ + \text{sen } 45^\circ &= 2 \text{sen } \frac{15^\circ + 45^\circ}{2} \cdot \cos \frac{15^\circ - 45^\circ}{2} = \\ &= 2 \text{sen } 30^\circ \cos (-15^\circ) \end{aligned}$$

$$\rightarrow \text{sen } p - \text{sen } q = 2 \cos \frac{p+q}{2} \text{sen } \frac{p-q}{2}$$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \text{sen } \alpha \text{sen } \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \text{sen } \alpha \text{sen } \beta \end{aligned}$$

$$\rightarrow \cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\rightarrow \cos p - \cos q = -2 \text{sen } \frac{p+q}{2} \text{sen } \frac{p-q}{2}$$

## WERNER

$$\operatorname{sen} \alpha \cos \beta = \frac{1}{2} [\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\operatorname{sen} \alpha \operatorname{sen} \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\operatorname{sen} 2\alpha \cdot \cos 3\alpha = \frac{1}{2} [\operatorname{sen} 5\alpha + \operatorname{sen}(-\alpha)] =$$

$$= \frac{1}{2} [\operatorname{sen} 5\alpha - \operatorname{sen} \alpha]$$